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Amdahl's Law and the statistical content of the NAS parallel benchmarks

The NAS Parallel Benchmarks have been developed at the NASA Ames Research Center. In the last three years extensive performance data have been reported for parallel machines both based on the NAS Parallel Benchmarks [1, 2] and on LINPACK [3]. In this study we have used the reported benchmark results and performed a number of statistical experiments. These included cluster, factor and regression analyses. We did this to find out how many of the NAS Parallel Benchmarks are - in a statistical sense - necessary, to represent all the reported results. We also fitted Amdahl's Law to the data, to see whether it is meaningful to apply more sophisticated performance models to the reported results. All statistical experiments were done for absolute performances as well as for the corresponding efficiencies. The analysis of Amdahl's Law was performed for both classes (Class A and B) of the NAS Parallel Benchmarks.

As parallel systems became more and more wide spread within the last years the interest in benchmark data of parallel systems increased. One of the best known and commonly used benchmarks in this area is the set of the NAS Parallel Benchmarks [1, 2]. This set of 8 "paper and pencil" benchmark problems has been developed at the NASA Ames Research Center. The latest results are available electronically on the WWW at the URL address: http://www.nas.nasa.gov/NAS/NPB/.

Another very common benchmark is the LINPACK benchmark [3], which has been in use for more than ten years. Results are also available elec-

tronically at the URL address:

http://www.netlib.org/benchmark/to-get-lp-benchmark. As performing complex benchmarks on a parallel system can be very time-consuming for the implementor, one might wonder how many of the 8 NAS PBs are necessary to describe and represent the data and the characteristics of the different systems and how many of them can be explained by the results of the others. In this study we try to find out whether it is possible to reduce the number of benchmarks without losing information, and which benchmarks are similar. We did this by factor analyses based on the correlation matrix between the benchmark

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5 Hockney, R.W. and C.R. Jesshope, *Parallel Computers 2*, Adam Hilger, Bristol, 1988.

6 Schönauer, W., Scientific Computer Computers, North-Holland, 1987.

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Schönauer, W. and H. Häfner, Explaining the gap between theoretical peak performance and real performance for supercomputer architectures, Scientific Programming 3, 157–168, 1994.



									·		
r_{\max}	0,99										
n_{\max}	0.87	0.82									
$n_{1/2}$	0.75	0.71	0.76								Marridge
EP	0.97	0.97	0.83	0.82			180X	× (1)			
MG	0.91	0.93	0.77	0.65	0.91	1114	1/2			M This	
CG	0.57	0.55	0.41	0.18	0.57	0.69			(41)	A 15	30 J. 1377
FT	0.85	0.90	0.53	0.64	0.87	0.95	0.77				
IS	0.65	0.65	0.52	0.25	0.64	0.78	0.99	0.84			
LU	0.94	0.94	0.90	0.65	0.90	0.99	0.75	0.96	0.84		
SP	0.96	0.96	0.91	0.76	0.95	0.99	0.70	0.96	0.79	0.99	0.00
BT	0.82	0.99	0.94	0.78	0.99	0.98	0.63	0.95	0.72	0.98	0.99
	$r_{ m peak}$	r_{max}	n_{max}	$n_{1/2}$	EP	MG	CG	FT	IS	LU	SP

Figure 2. The correlation matrix of the NAS PBs of class B and the LINPACK benchmark

also used the correlation matrix between the efficiencies of the benchmarks with respect to the peak performance to eliminate the strong effect of the overall correlation to peak performance (Figure 3 and Figure 4). On the average the individual correlations are now smaller but in general still high. The peak performance shows no big correlations (neither positive nor negative!) to any of the efficiencies of the benchmarks. The correlations between the parameter $n_{1/2}$ and the efficiencies are now negative and stronger compared to previous cases but still not as high as the benchmark correlations. This means that no general conclusions about the efficiencies of benchmarks can be made from the LINPACK parameters n_{max} and $n_{1/2}$.

IS LU SP BT	-0.25 -0.22 -0.19 0.06	0.79 0.79 0.78 0.81	-0.27 -0.21 -0.18 -0.07	-0.51 -0.58 -0.61 -0.38	0.29	0.86 0.87 0.85	0.86 0.91 0.61	0.83 0.88 0.73	0.84 0.91 0.77	0.95 0.83	0.75	
MG CG FT IS	-0.18 -0.22 -0.16 -0.25	0.78 0.70 0.74 0.79	-0.22 -0.32 -0.21 -0.27	-0.46 -0.59 -0.51 -0.51	0.42 0.57 0.55	0.78 0.89 0.85	0.88	0.93	0.04			
$r_{ m max}$ $n_{ m max}$ $n_{1/2}$ EP	-0.06 0.87 0.60 -0.14	-0.17 -0.55 0.46	0.66	-0.29	0.61							

Figure 3. The correlation matrix of the efficiencies of the NAS PBs of class A and the LINPACK benchmark results.

The factor analyses of the benchmarks

Factor analyses can be used as an explorative method to get an overview on the structure of the given data, but cannot be used for testing or proofing any hypothesis. Therefore much care must be taken in interpreting the results.

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results of the class A problem size. The result is checked by looking at linear regressions between different NAS PBs.

Amdahl's Law [4] gives a very simple model for the performance of a parallel system for different numbers of processors. We fitted the measured data to Amdahl's Law to see whether this is possible and whether sufficient statistical space for including additional parameters remains in this model. All analyses were done using the SAS statistical software package. The data used for the analyses in this paper is as of October 1994

The correlation matrices

As starting point for the factor analyses we had to calculate the correlation matrix. We used the NAS PB results of the class A benchmarks and the LINPACK results $r_{\rm max}$ from table 3 of the LINPACK report [3] for unlimited problem sizes. We included also the peak performance $r_{\rm peak}$, and the parameters $n_{\rm max}$ and $n_{1/2}$ from the LINPACK benchmark. This results in the matrices shown in Figure 1 for the class A problem sizes and in Figure 2 for the class B results.

r_{\max} n_{\max}	0.99 0.87 0.60	0.81	0.66							AVV.	
$n_{1/2}$	0.00	0.50	0.66								
EP	0.97	0.97	0.82	0.53							
MG	0.90	0.94	0.75	0.36	0.92						
CG	- 14 0	0.62	0.48	0.20	0.63	0.73					
FT		0.95	0.66	0.47	0.93	0.98	0.77				
IS	0.60	0.63	0.52	0.11	0.65	0.75	0.99	0.78			
LU	0.93	0.95	0.93	0.55	0.93	0.98	0.77	0.78	0.81		
SP	0.94	0.96	0.86	0.41	0.95	0.96	0.78	0.98		0.00	
BT	0.88	0.92	0.58	0.42	0.86	0.98	0.67	0.98	0.80 0.68	0.99 0.96	0.96
	$r_{ m peak}$	r_{max}	n_{max}	$n_{1/2}$	EP	MG	CG	FT	IS	LU	SP

Figure 1. The correlation matrix of the NAS PBs of class A and the LINPACK benchmark results.

You can see that the benchmark results and the peak performance are highly correlated in almost all cases except for CG and IS. The correlations between benchmark results and the parameters $n_{\rm max}$ and $n_{1/2}$ are on the average much smaller. Only $n_{\rm max}$ shows bigger correlations to some of the benchmarks. We found later on during our studies no evidence that these two parameters can be used to explain or determine benchmark results and did not include them in the later analyses.

The reason for the high correlations between benchmarks is the simple fact, that published benchmark results always improve with increasing system size. This is not very surprising as other results would not be published by vendors. These big correlations are the reason for problems in the factor analyses and their interpretation. They lead to a very dominating single factor which tends to hide all other effects. Therefore we



$r_{ m max}$ $n_{ m max}$ $n_{ m 1/2}$	0.99 0.87 0.75	0.82 0.71	0.76						arykila Roja Wi		
EP MG CG FT IS LU	0.97 0.91 0.57 0.85 0.65 0.94 0.96	0.97 0.93 0.55 0.90 0.65 0.94 0.96	0.83 0.77 0.41 0.53 0.52 0.90	0.82 0.65 0.18 0.64 0.25 0.65 0.76	0.91 0.57 0.87 0.64 0.90 0.95	0.69 0.95 0.78 0.99 0.99	0.77 0.99 0.75 0.70	0.84 0.96 0.96	0.84 0.79	v Ai	
SP BT	0.96	0.96	0.91	0.78	0.99	0.98	0.63	0.95	0.72	0.98	0.99
	$r_{ m peak}$	$r_{ m max}$	n_{max}	$n_{1/2}$	EP	MG	CG	FT	IS	LU	SP

Figure 2. The correlation matrix of the NAS PBs of class B and the LINPACK benchmark results.

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$r_{max} \ n_{max} \ n_{1/2}$	-0.06 0.87 0.60	-0.17 -0.55	0.66								
EP MG CG FT IS LU SP BT	-0.14 -0.18 -0.22 -0.16 -0.25 -0.22 -0.19 0.06	0.46 0.78 0.70 0.74 0.79 0.79 0.78 0.81	-0.15 -0.22 -0.32 -0.21 -0.27 -0.21 -0.18 -0.07	-0.29 -0.46 -0.59 -0.51 -0.58 -0.61 -0.38	0.61 0.42 0.57 0.55 0.29 0.35 0.44 EP	0.78 0.89 0.85 0.86 0.87 0.85 MG	0.88 0.93 0.86 0.91 0.61 CG	0.93 0.83 0.88 0.73 FT	0.84 0.91 0.77 IS	0.95 0.83 LU	0.75 SP

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LU	0.93	0.95	0.93	0.55	0.93	0.98	0.77	0.78	0.81		
SP	0.94	0.96	0.86	0.41	0.95	0.96	0.78	0.98	0.80	0.99	
BT	0.88	0.92	0.58	0.42	0.86	0.98	0.67	0.98	0.68	0.99	0.96
	$r_{ m peak}$	r_{max}	n_{max}	$n_{1/2}$	EP	MG	CG	FT	IS	LU	SP

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You can see that the benchmark results and the peak performance are highly correlated in almost all cases except for CG and IS. The correlations between benchmark results and the parameters $n_{\rm max}$ and $n_{1/2}$ are on the average much smaller. Only $n_{\rm max}$ shows bigger correlations to some of the benchmarks. We found later on during our studies no evidence that these two parameters can be used to explain or determine benchmark results and did not include them in the later analyses.

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6). We get eigenvalues of 6.2, 0.7 and 0.05 by extracting 3 factors. Factor 2 has high loadings from CG and IS and factor 1 from all other benchmarks. For factor 3 a safe interpretation cannot be made. None of the experiments with factor analyses ever showed an indication for more than four meaningful and independent factors in the group of nine benchmarks and the peak performance.

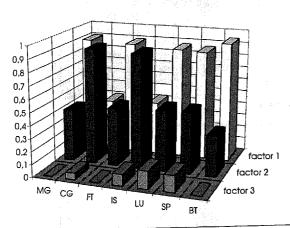


Figure 6. The loading of the 3 factors of the factor analysis for the subset of 7 NAS PBs only.

In addition to the performances we now analyze the correlations of the efficiencies in the same way. The result of the factor analysis of all benchmarks is shown in Figure 7. The first three eigenvalues of the correlation matrix are now 7.1, 0.9 and 0.5. Factor 3 shows a high loading from the efficiency of EP. Factor 1 and factor 2 show some different mix of the other benchmarks. Factor 1 has again high loadings from CG and IS which are not present in the other factors.

In Figure 8 the seven NAS PBs without EP show high loadings of CG and IS and in addition also of FT in factor 1. Factor 2 shows high loadings from MG and BT and factor 3 from LU and SP. Taking into consideration that FT was not loading high together with CG and IS in the case of the benchmark data we therefore summarize the factor analysis as follows:

- All benchmarks are strongly correlated with the peak performance. The different factor analyses indicate at the most four independent
- $r_{
 m max}$ from the LINPACK benchmark, EP and the peak performance are highly correlated and as a group form one factor of the analyses. - CG and IS as a group always form a second factor in the analyses.
- The remaining five NAS Parallel Benchmarks can be arranged in the two groups (LU and SP) and (MG, FT and BT). But the statistical

evidence for this splitting is not as clear as for the other groups. A common rule of experience demands at least 50 observations for applying a factor analysis at all. As the number of complete sets of mea-

$r_{ ext{max}}$ $n_{ ext{max}}$	-0.09 0.87 0.75	-0.22	0.56				J				4 N
$n_{1/2}$	0.75	-0.39	0.76				1				
EP MG	-0.22 -0.28	0.15 0.68	-0.21 -0.43	0.04 -0.42	0.13						tr Viti
CG	-0.25	0.70	-0.42	-0.49	0.31	0.89					
FT IS	-0.11 -0.30	0.48	-0.26	-0.31	0.36	0.74	0.82				
LU	-0.35	0.73 0.70	-0.44 -0.43	-0.52 -0.53	0.41 -0.03	0.87 0.95	0.99 0.90	0.82 0.67	0.88		
SP	-0.32	0.70	-0.48	-0.56	0.10	0.92	0.93	0.66	0.89	0.04	
BT	-0.07	0.63	-0.52	-0.57	0.51	0.86	0.89	0.69	0.89	0.94 0.93	0.96
	r _{peak}	r_{max}	n_{max}	$n_{1/2}$	EP	MG	CG	FΤ	IS	LU	SP

Figure 4. The correlation matrix of the efficiencies of the NAS PBs of class B and the LINPACK benchmark results.

First we applied a factor analysis to the correlation matrix of the benchmark results shown in Figure 1. The LINPACK parameters $n_{\rm max}$ and $n_{1/2}$ always came out as individual factors and thus gave us no additional information, so we did not include these two variables in the factor analyses any more. This first factor analysis gives a very dominant factor (with an eigenvalue of 8.8), which is related to the overall increase of benchmark performance with respect to the increase in peak performance. The next eigenvalues of the correlation matrix are 0.9 and 0.2 and thus for a rigid interpretation already quite small. Extracting these 3 factors and looking on the loading of their components after rotation (Figure 5) you can see that factor 2 has high loadings from CG and IS and factor 1 from EP, $r_{\rm max}$ and $r_{\rm peak}$, while factor 3 contains medium loadings from all other benchmarks.

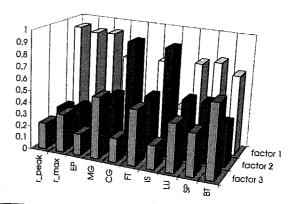


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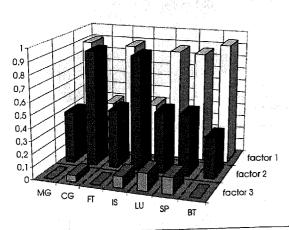


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$r_{max} \ n_{max}$	-0.09 0.87	-0.22		***************************************			,				7.88 4 2008
$n_{1/2}$	0.75	-0.39	0.76				/				
EP MG	-0.22 -0.28	0.15 0.68	-0.21 -0.43	0.04	0.10		*				41
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LU	-0.30 -0.35	0.73 0.70	-0.44 -0.43	-0.52 -0.53	0.41 -0.03	0.87 0.95	0.99 0.90	0.82 0.67	0.88		
SP	-0.32	0.70	-0.48	-0.56	0.10	0.92	0.93	0.66	0.89	0.94	
BT	-0.07	0.63	-0.52	-0.57	0.51	0.86	0.89	0.69	0.90	0.93	0.96
	$r_{ m peak}$	r_{max}	n_{max}	$n_{1/2}$	EP	MG	CG	FT	IS	LU	SP

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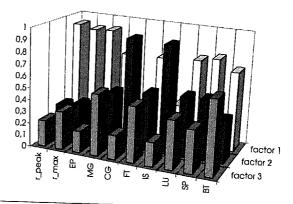


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gure 9. The slope of regression lines β 1 the R^2 values for pairwise regressions. values are shown in clower left corner of R^2 values in the per right. r_{peak} and max are measured Mflop/s while the AS PBs are given in AS PB units.

yses in the section above, we calculated the linear regressions between different pairs of benchmarks. As we never saw a statistical significance for an intercept term, we excluded it from the fit. Thus each regression is characterized by two parameters:

 $-\beta$: the slope of the regression line.

 $-R^2$: the portion of the variance σ explained by the regression. In the lower left of Figure 9 we show the slope β , in the upper right we show R^2 . All R^2 values are quite high. The regressions are better for pairs of benchmarks from within one of the identified groups than otherwise.

$\beta \backslash R^2$	r_{peak}	r_{max}	EP	MG	CG	FT	IS	LU	SP	ВТ
$r_{ m peak}$		0.998	0.990	0.950	0.426	0.936	0.464	0.884	0.923	0.990
	0.66333		0.993	0.963	0.458	0.951	0.497	0.903	0.941	0.996
EP	0.00152	0,00230		0.961	0.501	0.960	0.536	0.908	0.950	0.989
		0,00096				0.979	0.641	0.981	0.979	0.974
		0.00027				0.641	0.987	0.677	0.679	0.484
		0.00098			1.98		0.684	0.972		
75	0.00023	0.00036	0.163	0.42	1.26	0.42		0.734	0.721	0.527
		0,00055			1.18	0.57	0.97		0.978	0.926
		0.00095			1.99	0.97	1.62	1.67		0.958
					3.04	1.74	2.51	2.93	1.77	

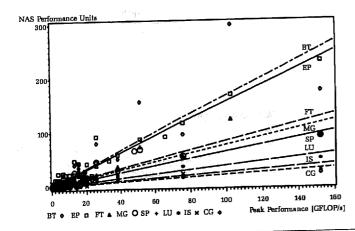


Figure 10. Linear egression of all NAS PBs versus the beak performance.

For a possible interpretation of the result of the factor analyses, we then plotted all NAS PB results over the peak performance (Figure 10) and

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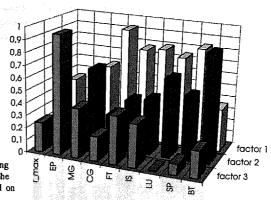


Figure 7. The loading of the 3 factors of the factor analysis based on efficiencies.

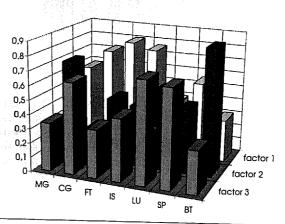


Figure 8. The loading of the 3 factors of the factor analysis for the subset of 7 NAS PBs only based on efficiencies.

surements for all benchmarks (about 30) is quite low compared to this, we calculated each coefficient of the correlation matrices by using also incomplete observations. This gives on the average 59 observations per element. But now the correlation matrices can have negative eigenvalues which might make a factor analysis meaningless. In our case the absolute values of the negative eigenvalues are very small and the dominating factors and their components are quite similar to the ones obtained by using complete observations only. So we take this as an additional confirmation of our analyses. For the class B problem size on the average only 30 observations are available for the correlation matrices. Due to this small statistical basis we do not report results of the factor analyses based on class B results. But at a first look they seem to be similar. For a first check of the groups of benchmarks identified by the factor analyses

yses in the section above, we calculated the linear regressions between different pairs of benchmarks. As we never saw a statistical significance for an intercept term, we excluded it from the fit. Thus each regression is characterized by two parameters:

 $-\beta$: the slope of the regression line.

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otherwise.

	$\beta \backslash R^2$	$r_{ m peak}$	r_{max}	EP	MG	CG .	FT	IS	LU	SP	ВТ
	$r_{ m peak}$		0.998	0.990	0.950	0.426	0.936	0.464	0.884	0.923	0.990
Figure 9. The slope of	r_{max}	0.66333		0.993	0,963	0.458	0.951	0.497	0.903	0,941	0.996
the regression lines β		0.00152	0.00230		0.961	0.501	0,960	0.536	0.908	0.950	0.989
and the R^2 values for all pairwise regressions.		0.00063				0.589				0.979	
β values are shown in		0,00018			0.32		0.641			0.679	
the lower left corner		0.00064				1.98		0.684		0.994	
and R ² values in the		0.00023				1.26	0.42		0.734	0.721	
upper right. rpeak and	LU	0.00036	0.00055	0.240	0.59	1.18	0.57	0.97		0.978	0.926
r _{max} are measured in Mflop/s while the		0.00063				1.99	0.97	1.62	1.67		0.958
NAS PBs are given in	вт	0.00117	0.00177	0.764	1.78	3.04	1.74	2.51	2.93	1.77	
NAS PB units.											

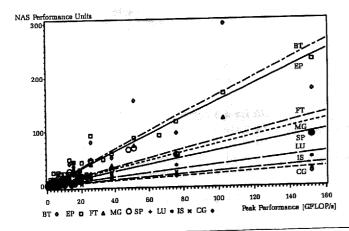


Figure 10. Linear regression of all NAS PBs versus the peak performance.

For a possible interpretation of the result of the factor analyses, we then plotted all NAS PB results over the peak performance (Figure 10) and

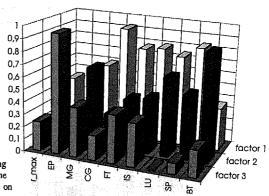


Figure 7. The loading of the 3 factors of the factor analysis based on efficiencies,

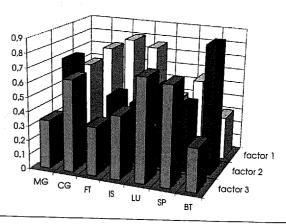


Figure 8. The loading of the 3 factors of the factor analysis for the subset of 7 NAS PBs only based on efficiencies.

surements for all benchmarks (about 30) is quite low compared to this, we calculated each coefficient of the correlation matrices by using also incomplete observations. This gives on the average 59 observations per element. But now the correlation matrices can have negative eigenvalues which might make a factor analysis meaningless. In our case the absolute values of the negative eigenvalues are very small and the dominating factors and their components are quite similar to the ones obtained by using complete observations only. So we take this as an additional confirmation of our analyses. For the class B problem size on the average only 30 observations are available for the correlation matrices. Due to this small statistical basis we do not report results of the factor analyses based on class B results. But at a first look they seem to be similar. For a first check of the groups of benchmarks identified by the factor anal-



gure 11. Single ocessor performances in NAS PB units tained by a fit Amdahl's Law all NAS PBs for ass A problem size. lissing values indicate easurements for only vo or less system

zes.

	r_1 of Class A	EP	MG	CG	FT	IS	ĽÜ	SP	BT
	CM2	0.00409	0.00103	0.00217	0.00185	0.00085	0.00121	0.00084	0.00151
	CM5	0.18736	0.03593	0.02024	0.09819			0.06697	0.10990
	CM5E	0.35747	0.19657	0.03644	0,17422	0.06013		0.09302	0.17805
	KSR1	0.07652	0.03702		gayanna ji		0.04660	0.03918	0.05632
	Meiko CS2	0.20828	0.20783	1.529	0.16815				ESE N
	nCube 2s	0.02351	0.00897	0.00633	0.00677	0.00764	0.00403	0.00538	0,00981
	SGI PowChal	0.51472		0.48834	0.51727		0.41571	0.55089	0.57377
	IBM SPI		0.15804	0.06619	0.08340	0.09351	0.13906	0.13248	0.21735
	IBM SP2	0.35536	0.43788	0.38369	0.21532	0.25259	0.36609	0.33067	0.41855
	SPP1000	0.32955	0,10930	0.06169	0.16566	0.14220	0.15734	0.18916	0.29769
	Cray T3D	0.22689	0.13125	0.04925	0.15273	0.08238	0.10443	0.13640	0.20387
	VPP500	2.78729	3.95379	2.25813	2,66994	4,99801		2.54733	5.07530
	Paragon XP	0.19190	0.05103	0.10621	0,07056		0.03230	0.04044	0.06887
•	Y-MP C90	2.72625	3.03040	3.45228	3.08355	3.58960	2.21656	2.41429	2.16728
	Y-MPel		0.30116	0,28088	0.31413	0.27584	0.22234	0.27992	0.23899
	α of Class A	EP	MG	CG	FT	IS	LU	SP	ВТ
	CM2	0.99988	0.99984	0.99937	0.99976	0.99939	0.99898	0.99934	0.99978
	CM5	0.99988	0.99780	0.99727	0.98578	0.99968	0.99281	0.99089	0.99122
	CM5E	0.99932	0,99592	0.99821	0.99039	0.99910	0.99092	0.99563	0.99697
	KSR1	1.00008	0.99856				0.98677	0.99421	0.99754
	Meiko CS2	0.99741	0.99439		0.98897				
	nCube 2s	1,00000	0.99984	0.99877	1.00001	0.99985	0.99935	0.99959	0.99973
	SGI PowChal	0.99930		0.91776	0.86025		0.98009	0.98286	0.99578
	IBM SP1		0.99498	0.98871	0.99700	0.99021	0.98916	0,98699	0.99262
	IBM SP2	1.00005	0.99583	0.95535	0.99733	0.99484	0.98805	0.99102	0.99445
	SPP1000	0.99894	0.96935	1.00027	0.97542		0.96209	0.96072	0.98246
	Cray T3D	0.99999	0.99950	0.99858	0.99926		0.99879	0.99941	0.99980
	VPP500	0.99901	0.97157	0.90730	0.99368	0.68633		0.94725	
			0.00704	0.97843	0.99517		0.99661	0.99551	0.99648
	Paragon XP	0.99985	0.99704	0.97643				0.00500	A 00000
	Paragon XP Y-MP C90		0.99704	0.96165	0.96262				
		0.99873			0.96262				
	Y-MP C90	0.99873	0.92711	0.96165 0.87243	0.96262	0,91003			

Figure 12. Parallelization ratios obtained by a fit of Amdahl's Law o all NAS PBs for ciass A problem size. Missing values indicate measurements for only two or less system sizes,

this case Amdahl's Law is too limited and cannot be extrapolated to unlimited processor numbers. So transformation of equation (2) fails for these cases.

The resulting parameters shown in Figures 11-14 give a good characterization and overview on the different systems and on the implementations of the benchmarks. For instance, is it quite easy to see extraordinarily good or bad implementations and results.

For the class B benchmark sizes we fitted the parameters shown in Figures 15-18.

In most cases Amdahl's Law fits very well to the data, giving small error bounds for any prediction typically in the range of a few percent. Thus

Con

to

calculated a linear regression over all results for different systems for each single benchmark. From top to bottom we found the ordering BT, EP, FT, MG, SP, LU, CG and IS. BT shows up higher than EP only because of the very well tuned results for the Fujitsu VPP500 system. So the different groups of benchmarks appear next to each other, giving a first interpretation of the results of the factor analyses: The different groups of benchmarks have different characteristic ranges of efficiencies of their implementations.

Applying Amdahl's Law to the NAS PBs

One of the simplest known models for the performance of a problem of fixed size on a parallel system is Amdahl's Law [4]. It's basic assumption is a split of the computational work in a sequential and in a fully parallelizable part. It can be characterized by the following parametrizations:

- α Fraction of parallelizable work in the implementation of the code
- $1-\alpha$ Fraction of sequential work in the implementation of the code
 - r_1 Performance running the code on a single processor given in units of the NAS PBs, which are dimensionless.
 - N Number of processors used
 - t_N Time for executing the program using N processors
- Sp_N Speedup of the program on N processors compared to one processor:

$$Sp_N = \frac{t_1}{t_N} = \frac{r_1 * N}{N - \alpha(N - 1)}$$
 (1)

A different parametrization can be obtained by introducing the asymptotic performance r_{∞} achieved by using an infinite number of processors and the processor number $N_{1/2}$ needed for achieving half of r_{∞} as follows:

$$r_{\infty} = \frac{r_1}{1-\alpha}$$
 and $N_{1/2} = \frac{\alpha}{1-\alpha}$ (2)

This gives:

$$Sp_N = \frac{r_\infty}{1 + \frac{N_{1/2}}{N}}$$

We fitted Amdahl's Law to the NAS PB to look whether this simple model for performance is already able to explain the measured performances or whether there is statistical room for more sophisticated models. As we wanted to calculate error terms we did this only for systems for which performance data of at least three different system sizes are reported. We allowed also α values greater than one, which does not make sense in a rigid application of Amdahl's Law. The results are shown in Figures 11 and 12 for the parametrization given in equation (1). By applying the transformation of equation (2) these values can be transformed into Figures 13 and 14. Some of the systems show α values slightly greater than one. This can be seen as an indication of superlinear speedups. In



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CM2 0.0049 0.00103 0.00217 0.00185 0.00085 0.00121 0.00840 0.0151 CM5 0.18736 0.83754 0.03593 0.02024 0.09819 0.02784 0.02894 0.06697 0.10900 CM5 0.07525 0.02874 0.19657 0.03644 0.17422 0.06013 0.08540 0.03030 0.17805 Meiko CS2 0.02851 0.08873 0.00633 0.06677 0.07646 0.04660 0.03918 0.05632 Figure 11. Single processor performances r1 in NAS PB units obtained by a fit of Amdahl's Law t0 all NAS PBs for class A problem size. 0.35955 0.15804 0.06619 0.16856 0.14200 0.13248 0.21735 SPP1000 0.32955 0.13926 0.1312 0.04569 0.15666 0.14220 0.15910 0.01619 0.16566 0.14220 0.15940 0.02936 0.03245 0.04924 0.03236 0.0414 0.03245 0.04924 0.04669 0.04669 0.04669 0.04669 0.04669 0.046687 0.046687 0.046687		r_1 of Class A	EP	MG	CG	FT Care	IS	LU	SP	BT
CM5 0.18736 0.03593 0.02024 0.09819 0.00778 0.02894 0.06697 0.109901 CM5E 0.35747 0.19657 0.03644 0.17422 0.06013 0.08540 0.09302 0.17805 KSR1 0.07652 0.02702 0.06813 0.06619 0.06619 0.04660 0.03918 0.05632 Meiko CS2 0.2028 0.02873 0.00633 0.06677 0.00764 0.0403 0.0538 0.09811 Figure 11. Single processor performances PL I I I MS PP 0.15804 0.06619 0.08340 0.09351 0.13906 0.13248 0.21735 IBM SP1 0.15804 0.6619 0.08340 0.09351 0.13906 0.13248 0.21735 IBM SP2 0.35536 0.43788 0.38369 0.15536 0.14220 0.15734 0.15104 0.03478 IBM SP1 0.32955 0.1930 0.06169 0.16566 0.14220 0.15734 0.13646 0.20387 IBM SP2 0.322689 0.13125 0.04925 0.15273 0.08238 0.10443 0.13640 0.20387 IBM SP2 0.32689 0.13125 0.04925 0.15273 0.08238 0.10443 0.13640 0.20387 IBM SP2 0.3910 0.05103 0.10621 0.07056 0.03230 0.04044 0.06887 Faragon XP 0.1919 0.05103 0.10621 0.07056 0.03230 0.04044 0.06887 Faragon XP 0.99988 0.99984 0.99937 0.99978 0.99988 0.99984 0.99938 IBM SP1 0.0006 0.99885 0.99937 0.99857 0.99998 0.99989 0.99989 IBM SP1 0.0006 0.99988 0.99877 0.0001 0.99981 0.99995 0.99957 IBM SP1 0.0006 0.99989 0.99877 0.0001 0.99985 0.99809 0.99978 IBM SP1 0.0006 0.99989 0.99877 0.0001 0.99980 0.99809 0.99978 IBM SP1 0.0006 0.99887 0.99977 0.98607 0.99809 0.99979 0.99978 IBM SP1 0.0006 0.99989 0.99887 0.99978 0.99980 0.99978 0.99978 0.99978 0.99980 0.99978 0.99980 0.99978 0.99978 0.99980 0.99978 0.99980 0.99978 0.99980 0.999		CM2	0.00409	0.00103	0.00217	0.00185	0.00085	0.00121	0.00084	0.00151
CMSE 0.35747 0.19657 0.03644 0.17422 0.06013 0.08540 0.09302 0.17805 KSR1 0.07652 0.03702 0.03705 0.03660 0.03918 0.05632 Meiko CS2 0.20828 0.20828 0.02878 0.06677 0.00764 0.00403 0.00538 0.00981 Figure 11. Single processor performances r1 in NAS PB units obtained by a fit of Amdahi's Law to all NAS PBs for class A problem size. Missing values indicate measurements for only two or less system sizes. Meiko CS2 0.23553 0.43788 0.38369 0.21532 0.12525 0.36609 0.33067 0.41855 Figure 11. Single processor performances r1 in NAS PB units obtained by a fit of Amdahi's Law to all NAS PBs for class A problem size. Missing values indicate measurements for only two or less system sizes. Meiko CS2 0.23559 0.02359 0.04025 0.04525 0.15273 0.08238 0.10443 0.13640 0.20387 Faragon XP 0.19190 0.05103 0.06169 0.16566 0.14220 0.15734 0.13640 0.20387 Faragon XP 0.19190 0.05103 0.06169 0.16566 0.16256 0.14220 0.15734 0.13640 0.20387 Faragon XP 0.19190 0.05103 0.10621 0.07056 0.03230 0.04044 0.06887 Faragon XP 0.99988 0.99988 0.99121 0.25888 0.31413 0.27584 0.22234 0.27992 0.23899 Faragon XP 0.99988 0.99980 0.99977 0.98878 0.99988 0.99981 0.99989 0.99989 Figure 11. Single professor performances r1 in Na Paragon XP 0.99990 0.99887 0.99887 0.99886 0.99887 0.99886 0.99887 0.99886 0.99887 0.99886 0.99887 0.99886 0.99887 0.99886 0.99886 0.99886 0.99886 0.99887 0.99886 0.99886 0.99886 0.99886 0.99886 0.99886 0.99886 0.99886 0.99886 0.99888 0.998									0.06697	0.10990
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Figure 11. Single processor performances r1 in NAS PB units obtained by a fit of Amdahl's Law to all NAS PBs for class A problem size. Missing values indicate measurements for only two or less system sizes. SPP1000 0.32955 0.10930 0.06169 0.16566 0.14220 0.15334 0.13640 0.290387 0.29769 0.39087 0.29769 0.39087 0.06189 0.16566 0.14220 0.15734 0.18916 0.290769 0.3018 0.100618 0.16566 0.14220 0.15734 0.13640 0.20387 0.20387 0.2038 0.2038 0.20387 0.2038 0.20						0.51727		0.41571		
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of Amdahl's Law to all NAS PBs for class A problem size. Missing values indicate measurements for only two or less system sizes. VPP500 2.78729 2.95813 2.25813 2.66994 4.99801 2.54733 5.07530 (2.54733 5.07530 2.25813) 2.25813 2.66994 4.99801 2.54733 5.07530 (2.54733 5.07530 2.25813) 2.25813 2.266994 4.99801 2.54733 5.07530 (2.54733 5.07530 2.25813) 2.25813 2.25813 2.266994 4.99801 2.54733 5.07530 (2.54733 5.07530 2.25813) 2.25813 2.266994 2.25813 2.					0.06169	0.16566	0.14220	0.15734		
to all NAS PBs for class A problem size. Missing values indicate measurements for only two or less system sizes. VPP500 Paragon XP Par						0.15273	0.08238	0.10443		
Paragon XP 0.19190 0.2888 0.19120 0.21656 0.21429 0.216728 0.21628				3,95379	2.25813	2.66994	4.99801			
measurements for only two or less system sizes. Y-MP C90 2.72625 3.03040 3.45228 3.08355 3.58960 2.21656 2.41429 2.16728 x-MP C90 2.7MPel 0.30116 0.28088 0.31413 0.27584 0.22234 0.27992 0.23899 cm c cm sizes. CM2 0.99988 0.99984 0.99937 0.99976 0.99939 0.99889 0.99934 0.99978 CM5 0.99988 0.99980 0.99972 0.98578 0.99968 0.99981 0.99089 0.99989 0.99978 CM5E 0.99932 0.99952 0.99821 0.99039 0.99969 0.99989 0.99978 KSR1 1.00008 0.99856 0.99897 0.99807 0.98677 0.99677 0.98677 0.998677 0.99867 0.99887 0.99975 0.99889 0.99975 0.99869 0.99888 0.99975 0.99809 0.99826 0.99878 0.99869 0.99935 0.99978 0.99978 0.99935 0.99935 0.99978 <td></td> <td>Paragon XP</td> <td>0.19190</td> <td>0.05103</td> <td>0.10621</td> <td>47 to 1 1 2 2 2</td> <td></td> <td></td> <td></td> <td></td>		Paragon XP	0.19190	0.05103	0.10621	47 to 1 1 2 2 2				
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0 000 40		CM2 CM5E KSR1 Meiko CS2 nCube 2s SGI PowChal IBM SP1 IBM SP2 SPP1000 Cray T3D VPP500	0.99988 0.99988 0.99932 1.00008 0.99741 1.00000 0.99930 1.00005 0.99894 0.99999 0.99901	0.99984 0.99780 0.99592 0.99856 0.99439 0.99984 0.99583 0.96935 0.99950 0.97157	0.99937 0.99727 0.99821 0.99877 0.91776 0.98871 0.95535 1.00027 0.99858 0.90730	0.99976 0.98578 0.99039 0.98897 1.00001 0.86025 0.99700 0.99733 0.97542 0.99926 0.99368	0.99939 0.99968 0.99910 0.99985 0.99021 0.99484 0.93409 0.99788 0.68633	0.99898 0.99281 0.99092 0.98677 0.99935 0.98009 0.98805 0.96209 0.99879	0.99934 0.99089 0.99563 0.99421 0.99959 0.98286 0.98699 0.99102 0.96072 0.99941 0.94725 0.99551	0.99978 0.99122 0.99697 0.99754 0.99973 0.99578 0.99262 0.99445 0.99246 0.99820 0.99857 0.99648
· · · · · · · · · · · · · · · · · · ·		CM2 CM5 CM5E KSR1 Meiko CS2 nCube 2s SGI PowChal IBM SP1 IBM SP2 SPP1000 Cray T3D VPP500 Paragon XP	0.99988 0.99988 0.99932 1.00008 0.99741 1.00000 0.99930 1.00005 0.99894 0.99990 0.99901 0.99985	0.99984 0.99780 0.99592 0.99856 0.99439 0.99984 0.99583 0.96935 0.99950 0.97157 0.99704	0.99937 0.99727 0.99821 0.99877 0.91776 0.98871 0.95535 1.00027 0.99858 0.90730 0.97843	0.99976 0.98578 0.99039 0.98897 1.00001 0.86025 0.99700 0.99733 0.97542 0.99926 0.99368 0.99517	0.99939 0.99968 0.99910 0.99985 0.99021 0.99484 0.93409 0.99788 0.68633	0.99898 0.99281 0.99092 0.98677 0.99935 0.98009 0.98805 0.96209 0.99879	0.99934 0.99089 0.99563 0.99421 0.99959 0.98286 0.98699 0.99102 0.96072 0.99941 0.94725 0.99551 0.99533	0.99978 0.99122 0.99697 0.99754 0.99973 0.99578 0.99262 0.99445 0.998246 0.99880 0.99857 0.99648 0.98383

Figure 12. Parallelization ratios α obtained by a fit of Amdahl's Law to all NAS PBs for class A problem size. Missing values indicate measurements for only two or less system sizes.

this case Amdahl's Law is too limited and cannot be extrapolated to unlimited processor numbers. So transformation of equation (2) fails for these cases.

The resulting parameters shown in Figures 11–14 give a good characterization and overview on the different systems and on the implementations of the benchmarks. For instance, is it quite easy to see extraordinarily good or bad implementations and results.

For the class B benchmark sizes we fitted the parameters shown in Figures 15–18.

In most cases Amdahl's Law fits very well to the data, giving small error bounds for any prediction typically in the range of a few percent. Thus

calculated a linear regression over all results for different systems for each single benchmark. From top to bottom we found the ordering BT, EP, FT, MG, SP, LU, CG and IS. BT shows up higher than EP only because of the very well tuned results for the Fujitsu VPP500 system. So the different groups of benchmarks appear next to each other, giving a first interpretation of the results of the factor analyses: The different groups of benchmarks have different characteristic ranges of efficiencies of their implementations.

Applying Amdahl's Law to the NAS PBs

One of the simplest known models for the performance of a problem of fixed size on a parallel system is Amdahl's Law [4]. It's basic assumption is a split of the computational work in a sequential and in a fully parallelizable part. It can be characterized by the following parametrizations:

- α Fraction of parallelizable work in the implementation of the code
- $1-\alpha$ Fraction of sequential work in the implementation of the code
 - r_1 Performance running the code on a single processor given in units of the NAS PBs, which are dimensionless.
 - N Number of processors used
 - t_N Time for executing the program using N processors
- Sp_N Speedup of the program on N processors compared to one processor:

$$Sp_N = \frac{t_1}{t_N} = \frac{r_1 * N}{N - \alpha(N-1)}$$
 (1)

A different parametrization can be obtained by introducing the asymptotic performance r_{∞} achieved by using an infinite number of processors and the processor number $N_{1/2}$ needed for achieving half of r_{∞} as follows:

$$r_{\infty} = \frac{r_1}{1-\alpha}$$
 and $N_{1/2} = \frac{\alpha}{1-\alpha}$ (2)

This gives:

$$Sp_N = \frac{r_\infty}{1 + \frac{N_{1/2}}{N}}$$

We fitted Amdahl's Law to the NAS PB to look whether this simple model for performance is already able to explain the measured performances or whether there is statistical room for more sophisticated models. As we wanted to calculate error terms we did this only for systems for which performance data of at least three different system sizes are reported. We allowed also α values greater than one, which does not make sense in a rigid application of Amdahl's Law. The results are shown in Figures 11 and 12 for the parametrization given in equation (1). By applying the transformation of equation (2) these values can be transformed into Figures 13 and 14. Some of the systems show α values slightly greater than one. This can be seen as an indication of superlinear speedups. In



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	r_1 of Class B	EP	MG	CG	F	T , , ,	IS	LU		SP	BT
	CM5E	0.12151	0.06228	0.0088	36 0	.06382	0.01313	0.06	5463	0.02204	0.04965
	Meiko CS2	0.07882	0.07026	nij ya		great da	P.C.				
Figure 15. Single	nCube 2s		To Resta	- 444	AŠP	all i	0.00203		La general	3 \$ 10 to 6 \$ \$ \$	
processor performances	SGI PowChal				0	.19526	Q ¹ 3	0.17	7161	0.21789	0.22499
r ₁ in NAS PB units obtained by a fit	IBM SP1		0.05711	0.0111	13 0	.02773	0.01875	0.0	7758	0.05184	
of Amdahl's Law	IBM SP2	0.14902	0.15836	0.0788	89 0	.07519	0.06636	0.17	7667	0.12340	
to all NAS PBs for		0.08263),04742	0.01757	0.04	4155	0.04172	
class B problem size.	VPP500	1.03391	1.39342	0.636	52 1	.04332				0.86942	
Missing values indicate measurements for only		0.06947	0.01743	0.009	20		0.01562			0.01278	
two or less system	Y-MP C90	1.00674	1.13505	0.994	27 2	2.65917	1.00900	1.0	3262	1.01801	1.0213
SIZES.	α of Class B	EP	MG	CG	F	т	IS	LU		SP	ВТ
		. 12122		1.0010	n n	.00000	0.99906	0.92	7627	0.99774	0.99783
		1.00020		1,0010	ט טע		0.33300	0.57		01227	
	Meiko CS2		0,99559				0.99996				and the
Figure 16.	nCube 2s				۸	.89185	0.77770	0.93	7924	0.96530	0.98557
Parallelization ratios	SGI PowChal	0.99947	0.99459	0.0069			1.00000		9042	0.99476	0.99746
α obtained by a fit	IBM SP1	1 00000	0.99439				0.99754		9415	0.99671	0.99809
of Amdahl's Law to all NAS PBs for		1.00009).99957	0.99932		9964	0.99949	0.99978
class B problem size.	Cray T3D	1.00000	0.97301).99777	0.,,,,,,,			0.97478	0,9996
Missing values indicate	VPP500		0.97301	0.998		,,,,,,,,,	0.99646	0.9	9745	0.99834	0.9982
measurements for only	Paragon XP Y-MP C90	0.99998	0.99771	0.550	ታ፥ ኃፈ የ	188526			7925	0.94467	0.9835
two or less system sizes.	Y-MP C90	0.99828	0.93933	0.57.5							
	r_{∞} of Class	B EF	MG	CG	FT	· IS	LU	SP	В	Т	
	CM5	E *	18.7	*	6.9	14,0	2.7	9.6	22	.9	
	Meiko CS	32 66.5	16.2								
	nCube 2	2s *	r			50.75					
	SGI PowCh	al 356.6	5		1.8		8.3	6.3		5.6	
	IBM SI	21	10.6	3.6	43.3		8,1	9.9		2.8	
	IBM SI	22	48.6	8.0	92.9	1.00		37.5		5.0	
	Сгау Т3	D '	101.4	777	110.3		115.4	81.8	333		
	VPP50	00 2297.0			505,4				5125	3.8	
	Paragon X			5.8		4.4		7.7		1.9	
	Y-MP C	90 585.	3 18.7	40.2	23.1	84.9	49.1	10.4	. 0.	1.7	

Figure 17. Asymptotic performances r_{∞} in NAS PB units obtained by a fit of Amdahl's Law to all NAS PBs for class B problem size. Missing values indicate measurements for only two or less system sizes. "*" denote entries for which $\alpha \ge 1$.

Conclusions

Applying factor analyses to the NAS PBs we found that at the most four factors can be extracted for which a meaningful interpretation is possible. They add up to more than 95% of the total variance of the input data. Hence four benchmarks are sufficient to characterize the overall NAS Parallel Benchmark performances. Looking at the individual factors resulting from the analyses the results can be summarized as follows:

- All benchmarks are strongly correlated with the peak performance.
- $r_{
 m max}$ from the LINPACK benchmark, EP and the peak performance

	r_{∞} of Class A	EP	MG	CG	F	r is	LU	SP	ВТ	21 (19ge)	
	CM2	34,1	6.4								
	CM5	1561.3		3.4				1.3	6.9	2000	ries Lies
	CM5E	526.0	16.3	6.8				7.4	12.5	i dige	e de la companya de l
	KSR1	320.0	48.2	20.4	18.1	66.8		21.3	58.8		
	Meiko CS2		25.7				3.5	6.8	22.9		
		80.4	37.0		15.2						10
	nCube 2s	*	1,6	5.1	*	50.5	6.2	13.1	36.3		777
	SGI PowChal	735.3		5.5	3.7		20.9	32.1	136.0		i.
	IBM SP1		31.5	5.9	22.8	9.6	12.8	10.2	29.3	(90) (13)	
	IBM SP2	*	105.0	8.6	80.6	49.0	28.1	36.8	75.4		
Figure 13. Asymptotic	SPP1000	310.9	3.6	*	6.7	2.2	4.2	4.8	17.0	11883	13.
performances rom in	Cray T3D	2268.9	262.5	34.7	205.8	38.9	86.3	231,2	1019,4	t Bayya	
NAS PB units obtained	VPP500	281.5	139,1	24.4	422.5	15.9		48.3	3549,2		į.
by a fit of Amdahl's	Paragon XP	1279,3	17.2	4.9	14.6		9.5	9.0	19.6	- Sal. 1	. 33
aw to all NAS PBs. *" denote entries for	Y-MP C90	2145.6	41.6	90.0	84.5	151.7	38.1	517.0	134.0	- 7590	À.
vhich α≥1.	Y-MPel		1.4	2.2	3.7	3.1	1.8	1.6	2.0	يار. دو	9
	$N_{1/2}$ of Class A	A E	P M	IG	CG	FT	IS	L	U SP	ВТ	÷
	CM	2 8332.	3 6249	0.0 1:	586.3	416.6	1683.3				
	CM:	5 8332.	3 453	.6 3	335.4	69.3	3124.0			112.9	
	CM5I	3 1469.	5 244	.1 5	557.7	103.1	1110.1			329.0	
	KSR	1 :	* 693					74.		405.5	
	Meiko CS2	385.	1 177	.3		89.7		/ T .	0 171.7	403.3	
	nCube 2:	s ,			21.0		6665.6	1537	5 2420 1	2702.7	
	SGI PowCha	1 1427.0			11.2	6.2	0,000	49.:		3702.7	
	IBM SPI		198.	.2	87.6	332.3	101.1			236.0	
	IBM SP2	: *			21.4	373.5	192.8	91.3		134.0	
	SPP1000				*	39.7		82.7		179.2	
	Cray T3D						14.2	25.4		56.0	
	VPP500					350.4	470.7	825.4		4999.0	
	Paragon XP				9.8	157.2	2.2		18.0	698.3	
	Y-MP C90					206.0		294.0		283.1	
	Y-MPel				25.1	25.1	41.2	16.2		60.8	
	I-MPel		3.	8	6.8	10.7	10.1	6.0	4.6	7.5	

Figure 14. Processor number $N_{1/2}$ necessary for achieving half of the asymptotic performance r_{∞} obtained by a fit of Amdahl's Law to all NAS PBs. "*" denote entries for which $\alpha \ge 1$.

no statistical space is left to include additional parameters in this model. This is not true in the case of the FT benchmark. We assume that this is due to the effect of parallelizing over a different number of dimensions of the FFT (1, 2 or 3 dimensions) for a different number of processors. If one fits only one unique Amdahl curve to the different domains of the implementation, this effect leads to a big error. Only by a closer look on the actual implementations, you would be able to decide if this explanation is true. Examples for the fitted curves for some systems and class A problem sizes are given in Figures 19–22. Here we also show as examples the error bounds for the fits of the BT and FT benchmarks. The errors for BT are typically quite small while the errors for FT are sometimes quite big.

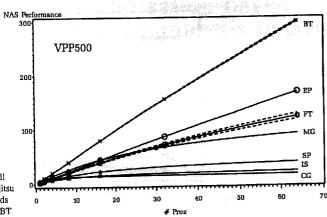


Figure 20. Fit of Amdahl's Law for all NAS PB for the Fujitsu VPP500. Error bounds are only shown for BT and FT.

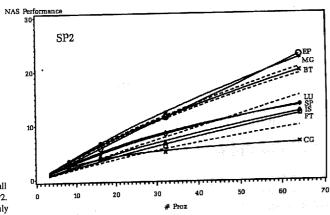


Figure 21. Fit of Amdahl's Law for all NAS PB for the SP2. Error bounds are only shown for BT and FT.

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Bailey, D.H., E. Barszcz, L. Dagum and H.D. Simon, NAS parallel benchmarks re-sults 10-94, NAS Technical Report RNR-94-001, NASA Ames Research Center, Moffett Field, CA 94035, October 1994.

are highly correlated and as a group form one factor of the analyses. - CG and IS as a group always form a second factor in the analyses.

- The remaining five NAS Parallel Benchmarks can be arranged in the two groups (LU and SP) and (MG, FT and BT). But the statistical evidence for this splitting is not as clear as for the other groups.

We also used Amdahl's Law to fit the measured performances. In most cases Amdahl's Law fits very well to the data, giving small error bounds for any prediction. The resulting parallelization ratios, single processor performances, asymptotic performances and $N_{1/2}$ processor numbers give a good characterization and overview on the different systems and on the implementations of the benchmarks. For a more detailed analysis you would need access to the implementations of the codes used by the different vendors, but unfortunately those are proprietary to the vendors,

Figure 18. Processor	$N_{1/2}$ of Class B	EP	MG	CG	FT	IS	LU	SP	вт
number $N_{1/2}$ necessary for achieving half of the asymptotic	CM5E Meiko CS2	* 839.3	299.3 231.6	*	107.7	1062.8	41.1	441.5	459.8
performance r_{∞}	nCube 2s	*				24991.0			
obtained by a fit	SGI PowChal	1885.8			8.2		47.2	27.8	68.3
of Amdahl's Law to all NAS PBs for	IBM SP1		183.9	319.5	1561.5	*	103.4	189.8	392.7
class B problem size.	IBM SP2	*	306.6	100.5	1233.6	405.5	169.9	303.0	522.6
Missing values indicate	Cray T3D	*	2126.7	1817.2	2324.6	1469.6	2776.8	1959.8	4544.5
measurements for only	VPP500	2221.2	36.1	20.3	483.3			38.7	2563.1
two or less system sizes. "*" denote	Paragon XP	49999,	435.7	627.9		281.5	391.2	601.4	583.8
entries for which $\alpha \ge 1$.	Y-MP C90	580.4	15.5	39.4	7.7	83.1	47.2	17.1	59.6

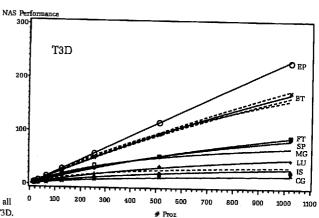


Figure 19. Fit of Amdahl's Law for all NAS PB for the T3D. Error bounds are only shown for BT and FT.

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Characterisation based bottleneck analysis of parallel systems

Bottleneck analysis plays an important role in the early design of parallel computers and programs. In this paper a methodology for bottleneck analysis based on an instruction level characterisation technique is presented. The methodology is based on the assumption that a bottleneck is caused by the slowest component of a computing system. These components are: memory (internal, external), processor (CPU, FPU), communication and I/O. Three metrics were used to identify bottlenecks in the system components. These are the B-ratio, the communication-computation ratio and the memory-processing ratio. These ratios are dimensionless and indicate the presence of a bottleneck when their values exceed unity. The methodology is illustrated and validated using a communication intensive linear solver algorithm (Gauss-Jordan elimination) which was implemented on a mesh connected distributed memory parallel computer (128 T800 Parsytec SuperCluster).

One of the main concerns of parallel computing is to port sequential programs efficiently knowing the resource limitations of the target machine such as processor, memory and communication network. In order to improve the performance of the parallel code bottleneck analysis is required. The identification of bottlenecks within parallel systems is an important aspect of hardware and software design. This process involves examining the system behavior under various load conditions. Bottlenecks can be defined in several ways as:

- The parts of the program that prevent achieving the optimal execution
- The parts of the system (either hardware or software) which consumes the maximum time or the slowest components of the system.

In this paper the second definition is used as the basis for the bottleneck analysis methodology which involves the following steps: predict the execution time components of a certain workload, identify the time component responsible for the bottleneck (the slowest part), analyze the component causing the bottleneck into its constituents and identify the sub-components causing the problem. Optimization of the software subroutines and/or hardware utilization causing the bottleneck can improve the system performance. This operation can be iterated until no further optimization is possible. Potential sources of bottlenecks are summarized

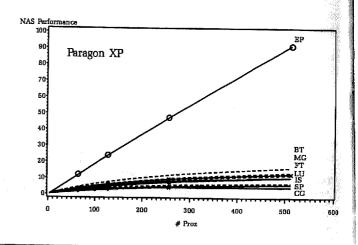


Figure 22. Fit of Amdahl's Law for all NAS PB for the Paragon XP running OSF1.2. Error bounds are only shown for BT and FT.

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